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Exercise sheet 3 to Advanced Partial Differential Equations

Exercise 1. Let $\Omega \subset \mathbb{R}^d$ be open and $\varphi \in W^{1,2}(\Omega)$.

i) Let $f \in C^1(\mathbb{R}^d)$, i.e., f is continuously differentiable and $\|f\|_{L^\infty(\mathbb{R}^d)} + \|Df\|_{L^\infty(\mathbb{R}^d;\mathbb{R}^d)} < \infty$. Show that $f \circ \varphi \in W^{1,2}(\Omega)$, and that the weak derivatives satisfy the chain rule.

ii) Show that $|\varphi| \in W^{1,2}(\Omega)$.

Exercise 2. Let $\Omega \subset \mathbb{R}^d$ be open and connected. Show that every $\varphi \in W^{1,2}(\Omega)$ satisfying $D\varphi = 0$ a.e. in Ω is in fact constant.

Hint: Show first by an approximation argument that φ is constant on each $K \subset\subset \Omega$ and conclude from there.

Exercise 3. Let $\Omega \subset \mathbb{R}^d$ be a C^1 domain and $l \in \mathbb{N}$. Show that $W_0^{l,2}(\Omega) = \{\varphi \in W^{l,2}(\mathbb{R}^d) \colon \operatorname{supp} \varphi \subset \overline{\Omega}\}$.

Hint: Try to mimic the core geometric idea of the proof of the approximation of Sobolev functions on Ω by restrictions of $\mathcal{D}(\mathbb{R}^d)$ functions; but instead of pushing the boundary to the outside, push it to the inside.